NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED

2021 Year 12 Course Assessment Task 4 (Trial HSC Examination)
Monday, 30 August 2021

General instructions

- Working time – 3 hour. (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

SECTION I

- Mark your answers on the answer grid provided (on page 27)

SECTION II

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: .... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 12MAA.1 – Miss Lee
◯ 12MAX.1 – Mr Ho
◯ 12MAX.2 – Mrs Bhamra
◯ 12MAX.3 – Mr Lam
◯ 12MAX.4 – Mr Sun

Marker's use only.

QUESTION	1-10	11-15	16-18	19-21	22-25	26-29	30-32	Total	%
MARKS	/10	/15	/14	/15	/16	/17	/13	/100	

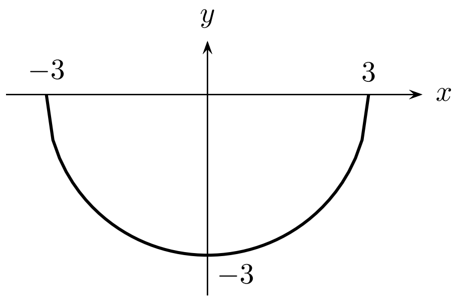
Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 27).

Questions **Marks**

1. The equation of the semi-circle illustrated below is given by **1**



(A) $y = \sqrt{9 - x^2}$

(B) $y = -\sqrt{9 - x^2}$

(C) $y = \sqrt{3 - x^2}$

(D) $y = -\sqrt{3 - x^2}$

2. The period of the function $f(x) = \sin\left(3x - \dfrac{\pi}{3}\right)$, where $x \in \mathbb{R}$, is **1**

(A) $\dfrac{\pi}{9}$

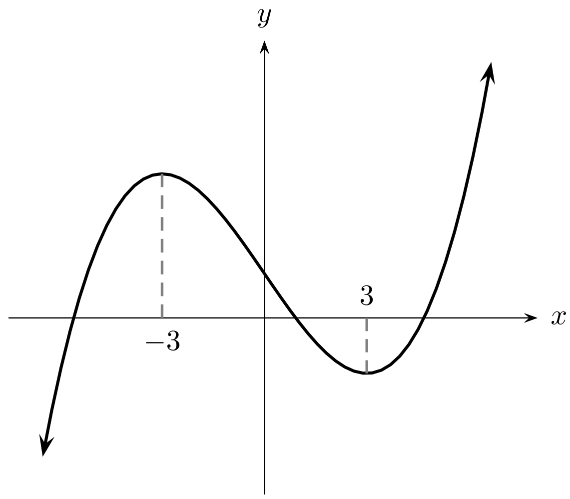
(B) $\dfrac{2\pi}{3}$

(C) 2π

(D) $\dfrac{\pi}{3}$

Examination continues overleaf…

3. The graph of $y = f(x)$ is shown below. For which interval(s) is $f'(x) < 0$? **1**



(A) $(-\infty, -3) \cup (3, \infty)$

(B) $(-3, 3)$

(C) $(-\infty, -3] \cup [3, \infty)$

(D) $[-3, 3]$

4. Which of the following is $e^{2\log_e x}$ equal to? **1**

(A) $\sqrt{x}$

(B) x^2

(C) $2x$

(D) e^{2x}

5. A particle, initially at rest at the origin, moves in a straight line with velocity $v = 6 - 2t$ m/s. **1**

What is the total distance travelled by the particle in the first 6 seconds?

(A) 0 m

(B) 6 m

(C) 9 m

(D) 18 m

Examination continues overleaf...

6. If $\int_1^4 f(x)\,dx = 2$, which of the following is $\int_1^4 (2f(x)+3)\,dx$ equal to? **1**

(A) 2

(B) 13

(C) 7

(D) 10

7. In a class of 30 girls, 13 are dancers and 23 are gymnasts. If 7 girls do both dance and gymnastics, what is the probability that a girl chosen at random does neither dance nor gymnastics? **1**

(A) $\frac{1}{5}$

(B) $\frac{7}{30}$

(C) $\frac{1}{30}$

(D) $\frac{8}{15}$

8. The circle $x^2 + y^2 - 4x - 4y + c = 0$ touches both the x and y axes. What is the value of c? **1**

(A) $c = 1$

(B) $c = 2$

(C) $c = 4$

(D) $c = 8$

9. The amount of water that Eleanor uses to wash her car is normally distributed with a mean of 50 litres and a standard deviation of 4 litres. **1**
On what percentage of occasions would Eleanor expect to use between 42 litres and 46 litres of water to wash her car?

(A) 13.5%

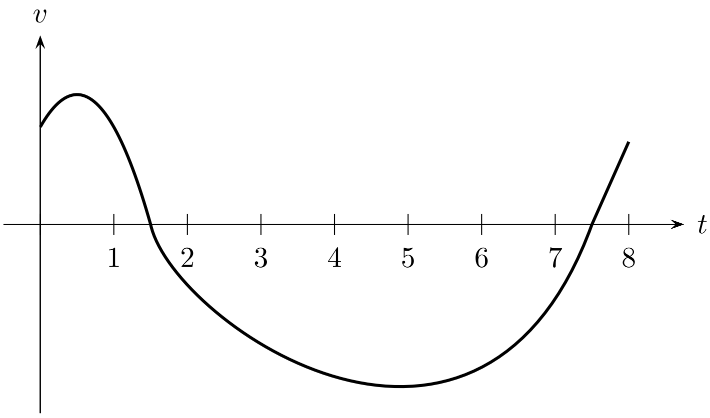
(B) 27%

(C) 34%

(D) 68%

Examination continues overleaf...

10. A particle is moving along the x-axis. The graph shows its velocity v metres per second at time t seconds. **1**



Initially, the particle is at the origin.

Which of the following describes the position of the particle after 8 seconds?

(A) The particle is to the left of the origin.

(B) The particle is to the right of the origin.

(C) The particle is at the origin.

(D) There is not enough information to determine its position.

Examination continues overleaf...

Section II

90 marks
Attempt Question 11 to 32
Allow approximately 2 hours and 45 minutes for this section

Write your answers in the space provided.

Question 11 (2 Marks)

Consider the polynomial $P(x) = (x+1)(x-1)(2x-5)$.

(a) State the degree of $P(x)$. **1**

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(b) State whether $P(x)$ is one-to-one, one-to-many, many-to-one or many-to-many. **1**

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Question 12 (3 Marks)

Differentiate the following:

(a) $y = (3x^2 - x + 1)^5$ **2**

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(b) $y = 2e^{x^2+x+1}$ **1**

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Examination continues overleaf...

Question 13 (4 Marks)

Find the following:

(a) $\displaystyle\int \left(x + \frac{1}{x}\right) dx$ **2**

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(b) $\displaystyle\int xe^{x^2+1}\, dx$ **2**

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Question 14 (3 Marks)

(a) Differentiate $\dfrac{x}{1+\ln x}$. **2**

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(b) Hence, or otherwise, find the primitive of $\dfrac{\ln x}{(1+\ln x)^2}$. **1**

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Examination continues overleaf...

Question 15 (3 Marks)

Find the equation of the normal to $y = x \sin x$ at the point where $x = \dfrac{\pi}{2}$. **3**

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Examination continues overleaf...

Question 16 (5 Marks)

The number N of bacteria in a mouldy loaf of bread at time t hours is given by the equation

$$N = 21e^{kt}$$

After 7 hours, the number of bacteria present is 30.

(a) Find the exact value of k. **2**

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(b) Determine the number of bacteria after 24 hours. **1**

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(c) At what rate is the number of bacteria increasing after 24 hours? Give your answer to 1 decimal place. **2**

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Examination continues overleaf...

Question 17 (6 Marks)

The acceleration of a moving body is given by $a = \sqrt{2t+1}$ ms^{-2}.

(a) If the body starts from rest, find its velocity after 4 seconds. **3**

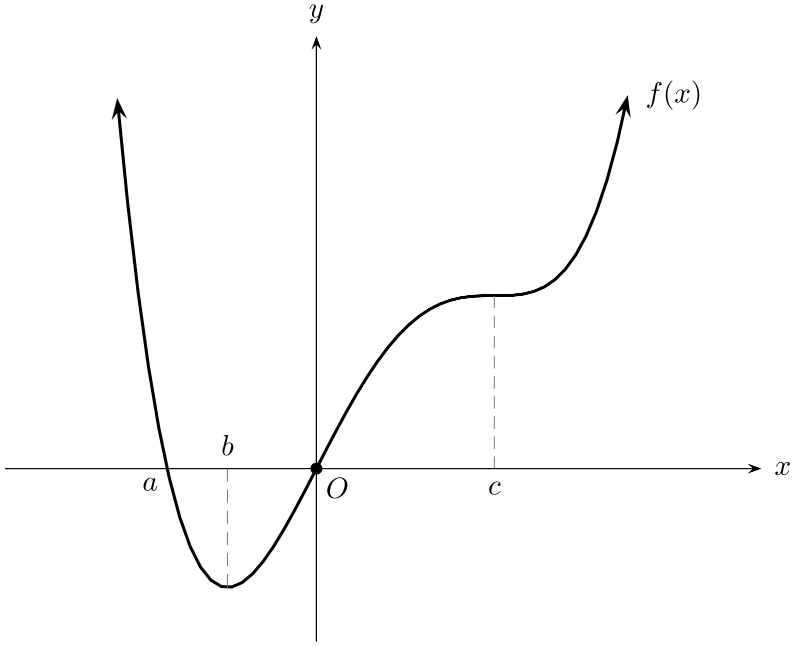
(b) Find, in exact form, the average velocity of the body during the fourth second. **3**

Examination continues overleaf...

Question 18 (3 Marks)

The graph of $f(x)$ is shown below. There is a minimum turning point at $x = b$, an inflexion point at O, and a horizontal point of inflexion at $x = c$. **3**

Draw the graph of $f'(x)$ on the same set of axes below.



Examination continues overleaf...

Question 19 (5 Marks)

(a) Use the trapezoidal rule with 5 function values to find an approximation to the value of $\int_1^5 \ln x \, dx$, correct to 4 decimal places. **3**

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(b) With the aid of a sketch, explain whether the approximation found in part (a) is an over-approximation or an under-approximation. **2**

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Examination continues overleaf...

Question 20 (5 Marks)

A university student plans to finish university in two years time. He wishes to travel overseas and thinks he will need \$15000. He decides to invest \$$M$ at the start of each month in an investment account earning 3% per annum compounded monthly.
Let A_n be the amount at the end of n months.

(a) Find A_2 in terms of M. **1**

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(b) Show that $A_n = 401M(1.0025^n - 1)$. **2**

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(c) Find the amount he must invest each month to reach his target of \$15000. Give your answer to the nearest dollar. **2**

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Examination continues overleaf...

Question 21 (5 Marks)

Sketch the graph of the curve $y = x^4 + 2x^3 + 1$ for $-2 \leq x \leq 1$, labelling the stationary points and points of inflexion. Also state the global maximum in the specified domain. **5**

(Do NOT determine the x intercepts of the curve.)

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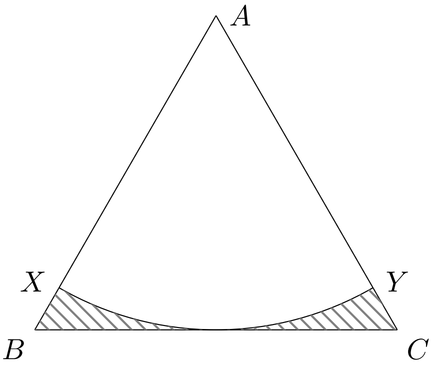
Examination continues overleaf. . .

Question 22 (6 Marks)

In the diagram, $\triangle ABC$ is an equilateral triangle with sides of length 6 cm. An arc with centre A and BC as tangent, cuts AB and AC at X and Y respectively.



(a) Show that the radius of the arc is $3\sqrt{3}$ cm. **2**

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(b) Find the exact length of arc XY. **1**

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(c) Find in exact form, the area of the shaded region. **3**

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Examination continues overleaf...

Question 23 (2 Marks)

Find the value(s) of $\tan x$ when $\tan^2 x + \sec^2 x = 9$. **2**

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Question 24 (2 Marks)

Prove that $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$. **2**

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Examination continues overleaf...

Question 25 (6 Marks)

A game consists of a player tossing a fair coin three times. You lose \$3.00 if three heads appear and lose \$2.00 if two heads appear. You win \$1.00 if one head appears and win \$3.00 if no heads appear. Let X be the amount you win or lose per game.

(a) Complete the probability distribution table below. **2**

x	-3	-2	1	3
$P(X = x)$				

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(b) Calculate $E(X)$ and hence determine the expected profit or loss if you played the game 1000 times. **2**

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(c) Evaluate $P(X > -2 | X \geq -2)$. **2**

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Examination continues overleaf...

Question 26 (4 Marks)

A continuous random variable T, represents the time taken in days to show symptoms after contracting a virus. It has the following probability density function

$$f(t) = \begin{cases} \dfrac{k}{2t-1} & \text{for } 1 \le t \le 14 \\ \\ 0 & \text{for } 0 \le t < 1 \text{ or } t > 14 \end{cases}$$

(a) Show that $k = \dfrac{2}{\ln 27}$. **2**

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Examination continues overleaf. . .

(b) After how many days will a person have a 75% chance of showing symptoms after they have contracted the virus? **2**

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Examination continues overleaf…

Question 27 (3 Marks)

Give a possible sequence of transformations that transforms $y = \dfrac{1}{x}$ to $y = \dfrac{3}{1-2x} + 5$. **3**
Ensure you use appropriate terminology in your response.

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Examination continues overleaf...

Question 28 (6 Marks)

(a) Sketch the graph of $y = |2x - 3|$ on the lines provided below. Label the x and y intercepts. **2**

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(b) Hence, or otherwise, solve $|6x - 9| + 5 = 8$. **3**

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(c) By using the graph sketched in part (a), or otherwise, state the values of m such that $|2x - 3| = mx$ has exactly 1 real solution. **1**

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Examination continues overleaf...

Question 29 (4 Marks)

Suppose $f(x) = \dfrac{1}{x^2 + x - 2}$ and $g(x) = \sin x$. **4**

Find the vertical asymptotes of $y = f\left(g(x)\right)$ within the domain $[0, 2\pi]$.

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Examination continues overleaf…

Question 30 (5 Marks)

There are 4 red and 3 black discs in a bag. Sophie and Emma are playing a game in which they take turns drawing a disc from the bag and then replacing it.

To win the game, Sophie must draw a red disc and for Emma to win she must draw a black disc. They continue taking turns until there is a winner. Sophie goes first.

(a) Find the probability that Sophie wins on her first draw. **1**

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(b) Find the probability that Sophie wins in three or less of her turns. **2**

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(c) Find the probability that Sophie wins the game. **2**

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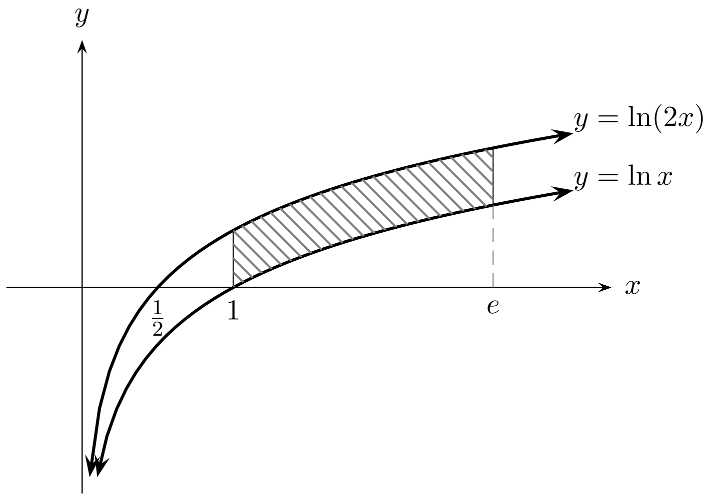
Examination continues overleaf...

Question 31 (3 Marks)

The curves of $y = \ln x$ and $y = \ln(2x)$ are drawn below. **3**



Find the shaded area, in exact form, between the curves $y = \ln(2x)$ and $y = \ln x$ and the lines $x = 1$ and $x = e$.

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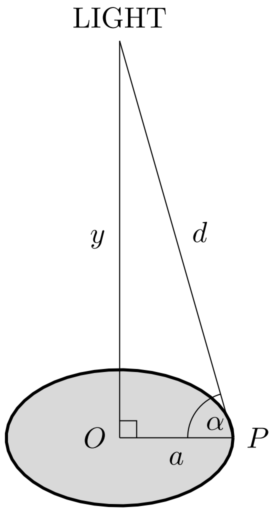
Examination continues overleaf…

Question 32 (5 Marks)

A light is to be placed over the centre of a circle, radius a units.
The intensity, I, of the light is proportional to the sine of the angle, α, at which the rays strike the circumference of the circle, divided by the square of the distance, d, from the light to the circumference of the circle; i.e. $I = \dfrac{k \sin \alpha}{d^2}$, where k is a constant.



(a) Show that $I = \dfrac{ky}{(y^2 + a^2)^{\frac{3}{2}}}$. **2**

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Examination continues overleaf...

(b) Find the best height for the light to be placed over the centre of the circle so as to provide the maximum illumination to the circumference. **3**

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End of paper.

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

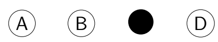
NESA STUDENT #: ....

Class (please ✔)

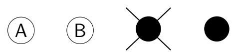
- ◯ 12MAA.1 – Miss Lee
- ◯ 12MAX.1 – Mr Ho
- ◯ 12MAX.2 – Mrs Bhamra
- ◯ 12MAX.3 – Mr Lam
- ◯ 12MAX.4 – Mr Sun

Directions for multiple choice answers

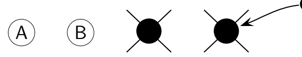
- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.



- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:



1 –	Ⓐ	Ⓑ	Ⓒ	Ⓓ	**6** –	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2 –	Ⓐ	Ⓑ	Ⓒ	Ⓓ	**7** –	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3 –	Ⓐ	Ⓑ	Ⓒ	Ⓓ	**8** –	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4 –	Ⓐ	Ⓑ	Ⓒ	Ⓓ	**9** –	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5 –	Ⓐ	Ⓑ	Ⓒ	Ⓓ	**10** –	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Sample Band 6 Responses

Section I

1. (B) **2.** (B) **3.** (B) **4.** (B) **5.** (D)
6. (B) **7.** (C) **8.** (C) **9.** (A) **10.** (A)

Section II

Question 11 (Bhamra)

(a) (1 mark)

✓ [1] for correct degree

Degree = 3.

(b) (1 mark)

✓ [1] for correct classification

$P(x)$ is many-to-one.

Question 12 (Bhamra)

(a) (2 marks)

✓ [1] for obtaining $5(3x^2-x+1)^4$

✓ [1] for correct application of chain rule

$$\begin{aligned}\frac{dy}{dx} &= 5(3x^2-x+1)^4 \times (6x-1)\\ &= 5(6x-1)(3x^2-x+1)^4\end{aligned}$$

(b) (1 mark)

✓ [1] for correct derivative

$$\frac{dy}{dx} = 2(2x+1)e^{x^2+x+1}$$

Question 13 (Bhamra)

(a) (2 marks)

✓ [1] for correct integration of x OR $\frac{1}{x}$

✓ [1] for correct primitive (including $+C$)

$$\int \left(x+\frac{1}{x}\right)dx = \frac{1}{2}x^2 + \ln x + C$$

(b) (2 marks)

✓ [1] for obtaining an equivalent integral with $2x$

✓ [1] for correct primitive (including $+C$)

$$\begin{aligned}\int xe^{x^2+1}dx &= \frac{1}{2}\int 2xe^{x^2+1}dx\\ &= \frac{1}{2}e^{x^2+1} + C\end{aligned}$$

Question 14 (Bhamra)

(a) (2 marks)

✓ [1] for correctly applying the quotient rule

✓ [1] for correct, fully simplified, derivative

$$\begin{aligned}\frac{d}{dx}\left(\frac{x}{1+\ln x}\right) &= \frac{(1+\ln x)(1) - x(\frac{1}{x})}{(1+\ln x)^2}\\ &= \frac{\ln x}{(1+\ln x)^2}\end{aligned}$$

(b) (1 mark)

✓ [1] for correct primitive (including $+C$)

$$\int \frac{\ln x}{(1+\ln x)^2}dx = \frac{x}{1+\ln x} + C$$

Question 15 (Bhamra) (3 marks)

✓ [1] for correct derivative

✓ [1] for finding gradient of normal at $x = \frac{\pi}{2}$

✓ [1] for correct equation of normal

$$\frac{dy}{dx} = x\cos x + \sin x$$

At $x = \dfrac{\pi}{2}$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\\ &= 1\\ \therefore m_N &= -1\end{aligned}$$

$\therefore$ Equation of normal at $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$:

$$\begin{aligned}y - \frac{\pi}{2} &= -\left(x - \frac{\pi}{2}\right)\\ \therefore y &= -x + \pi\end{aligned}$$

Question 16 (Ho)

(a) (2 marks)

✓ [1] for obtaining $30 = 21e^{7k}$

✓ [1] for correct exact value of k

When $t = 7$, $N = 30$

$$\begin{aligned} \therefore 30 &= 21e^{7k} \\ e^{7k} &= \frac{10}{7} \\ 7k &= \ln\frac{10}{7} \\ \therefore k &= \frac{1}{7}\ln\frac{10}{7} \end{aligned}$$

(b) (1 mark)

✓ [1] for correct value of N (rounding errors are ignored)

When $t = 24$,

$$\begin{aligned} N &= 21e^{24k} \\ &= 71.336... \\ &\approx 71 \text{ bacteria} \end{aligned}$$

(c) (2 marks)

✓ [1] for correct expression of $\dfrac{dN}{dt}$

✓ [1] for correct rate when $t = 24$ (rounding errors are ignored)

$$\frac{dN}{dt} = 21ke^{kt}$$

When $t = 24$,

$$\begin{aligned} \frac{dN}{dt} &= 21ke^{24k} \\ &= 3.634... \\ &\approx 3.6 \text{ bacteria/hr} \end{aligned}$$

Question 17 (Ho)

(a) (3 marks)

- ✓ [1] for obtaining $v = \dfrac{(2t+1)^{\frac{3}{2}}}{3} + C$
- ✓ [1] for finding the value of C
- ✓ [1] for finding v when $t = 4$

$$\begin{aligned} a &= (2t+1)^{\frac{1}{2}} \\ \therefore v &= \int (2t+1)^{\frac{1}{2}}\,dt \\ &= \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + C \\ &= \frac{(2t+1)^{\frac{3}{2}}}{3} + C \end{aligned}$$

When $t = 0$, $v = 0$

$$\begin{aligned} \therefore 0 &= \frac{1}{3} + C \\ \therefore C &= -\frac{1}{3} \\ \therefore v &= \frac{(2t+1)^{\frac{3}{2}}}{3} - \frac{1}{3} \end{aligned}$$

When $t = 4$,

$$\begin{aligned} v &= \frac{9^{\frac{3}{2}}}{3} - \frac{1}{3} \\ &= \frac{27}{3} - \frac{1}{3} \\ &= \frac{26}{3}\ \text{m/s} \end{aligned}$$

(b) (3 marks)

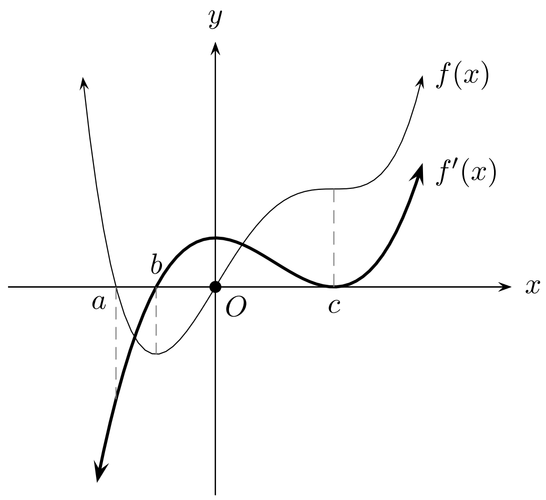
- ✓ [1] for correct interpretation of 'during the fourth second'
- ✓ [1] for obtaining $v_{\text{avg}} = \int_3^4 v(t)\,dt$
- ✓ [1] for finding the exact average velocity

During the fourth second means between $t = 3$ to $t = 4$.

$$\begin{aligned} \therefore v_{\text{avg}} &= \frac{x(4) - x(3)}{4 - 3} \\ &= x(4) - x(3) \\ &= \int_3^4 v(t)\,dt \\ &= \int_3^4 \frac{(2t+1)^{\frac{3}{2}}}{3} - \frac{1}{3}\,dt \\ &= \frac{1}{3}\int_3^4 (2t+1)^{\frac{3}{2}} - 1\,dt \\ &= \frac{1}{3}\left[\frac{(2t+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2} - t\right]_3^4 \\ &= \frac{1}{3}\left[\frac{(2t+1)^{\frac{5}{2}}}{5} - t\right]_3^4 \\ &= \frac{1}{3}\left(\frac{223}{5} - \left(\frac{7^{\frac{5}{2}}}{5} - 3\right)\right) \\ &= \frac{1}{3}\left(\frac{238}{5} - \frac{7^{\frac{5}{2}}}{5}\right)\ \text{m/s} \end{aligned}$$

Question 18 (Ho) (3 marks)

- ✓ [1] for correct overall shape of $f'(x)$
- ✓ [1] for correct location of $f'(x)$ at either O, $x = b$, OR $x = c$
- ✓ [1] for correct locations of $f'(x)$ at O, $x = a$, $x = b$, AND $x = c$



Question 19 (Lee)

(a) (3 marks)

✓ [1] for correct x-values

✓ [1] for correct y-values

✓ [1] for final answer (rounding errors are ignored)

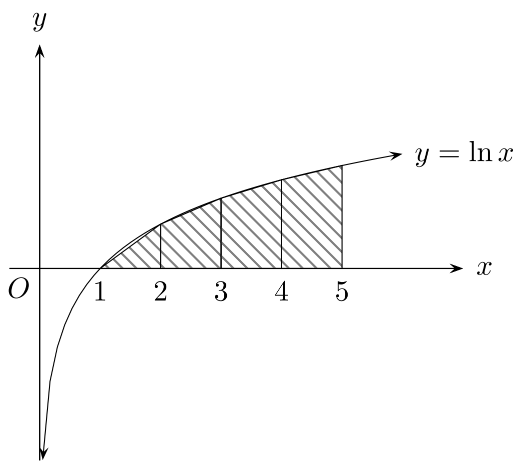
x	1	2	3	4	5
y	0	$\ln 2$	$\ln 3$	$\ln 4$	$\ln 5$

$$\begin{aligned}\therefore \int_1^5 \ln x dx &\approx \frac{1}{2}\left[\ln 5 + 2(\ln 2 + \ln 3 + \ln 4)\right] \\ &= 3.98277... \\ &= 3.9828 \text{ (4 d.p.)}\end{aligned}$$

(b) (2 marks)

✓ [1] for sketching $y = \ln x$ and trapezia between $x = 1$ to $x = 5$

✓ [1] for correct explanation



As the trapezia lie entirely beneath the graph of $y = \ln x$, the approximation found in part (a) is an under-approximation of $\displaystyle\int_1^5 \ln x dx$.

OR

As the trapezia do not completely cover the area represented by $\displaystyle\int_1^5 \ln x dx$, the approximation found in part (a) is an under-approximation.

OR

Any valid explanation.

Question 20 (Lee)

(a) (1 mark)

✓ [1] for correct expression of A_2

$$A_1 = M(1.0025)$$

$$\begin{aligned} A_2 &= [M(1.0025) + M](1.0025) \\ &= M(1.0025)^2 + M(1.0025) \end{aligned}$$

(b) (2 marks)

✓ [1] for obtaining correct expression of A_3

✓ [1] for correct proof

$$\begin{aligned} A_3 &= [M(1.0025)^2 + M(1.0025) + M](1.0025) \\ &= M(1.0025) + M(1.0025)^2 + M(1.0025)^3 \end{aligned}$$

$$\begin{aligned} \therefore A_n &= M(1.0025) + M(1.0025)^2 + M(1.0025)^3 + \cdots + M(1.0025)^n \\ &= M\left(\frac{1.0025(1.0025^n - 1)}{1.0025 - 1}\right) \\ &= 401M(1.0025^n - 1) \end{aligned}$$

(c) (2 marks)

✓ [1] for substituting $n = 24$ and $A_{24} = 15000$

✓ [1] for correct value of M (no mark if M is rounded incorrectly)

When $n = 24,\ A_n = \$15000$

$$\begin{aligned} \therefore 15000 &= 401M(1.0025^{24} - 1) \\ \therefore M &= 605.703... \end{aligned}$$

Therefore, he must invest \$606 per month.

Question 21 (Lee) (5 marks)

- ✓ [1] for determining the x values of the stationary points
- ✓ [1] for determining the nature of ALL stationary points
- ✓ [1] for determining the points of inflection
- ✓ [1] for correct sketch within the given domain, with ALL features labelled
- ✓ [1] for correct global maximum value

$$\frac{dy}{dx} = 4x^3 + 6x^2$$

Let $\frac{dy}{dx} = 0$

$$\begin{aligned} \therefore 4x^3 + 6x^2 &= 0 \\ 2x^2(2x+3) &= 0 \\ \therefore x &= 0, -\frac{3}{2} \end{aligned}$$

$$\frac{d^2y}{dx^2} = 12x^2 + 12x$$

When $x = 0$, $\frac{d^2y}{dx^2} = 0 \implies$ inconclusive; use table

x	-1	0	1
$f'(x)$	2	0	10
	↗	⟶	↗

$\therefore (0, 1)$ is a horizontal point of inflexion.

When $x = -\frac{3}{2}$, $\frac{d^2y}{dx^2} = 9 > 0 \implies \left(-\frac{3}{2}, -\frac{11}{16}\right)$ is a minimum turning point.

Let $\frac{d^2y}{dx^2} = 0$

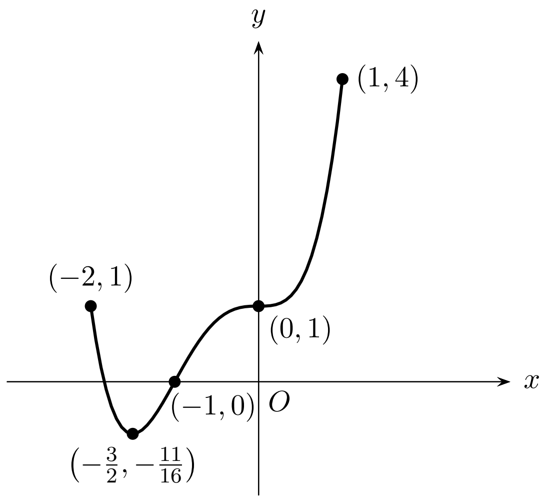
$$\begin{aligned} \therefore 12x^2 + 12x &= 0 \\ 12x(x+1) &= 0 \\ \therefore x &= 0, -1 \end{aligned}$$

Already established there is a point of inflection at $x = 0$.

Check whether there is a point of inflection at $x = -1$:

x	-2	-1	-0.5
$f''(x)$	24	0	-3
	$+$		$-$

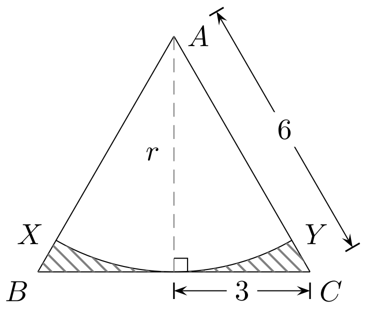
$\therefore (-1, 0)$ is a point of inflection.



From the graph, global maximum $= 4$.

Question 22 (Sun)

(a) (2 marks)
- ✓ [1] for substantial progress
- ✓ [1] for correct proof



$$\begin{aligned} r &= \sqrt{6^2 - 3^2} \\ &= \sqrt{27} \\ &= 3\sqrt{3} \text{ cm} \end{aligned}$$

(b) (1 mark)
- ✓ [1] for final answer

$$\begin{aligned} \text{arc } XY &= 3\sqrt{3} \times \frac{\pi}{3} \\ &= \sqrt{3}\pi \text{ cm} \end{aligned}$$

(c) (3 marks)
- ✓ [1] for correct value of $A_\triangle$
- ✓ [1] for correct value of A_{sector}
- ✓ [1] for final answer

$$\begin{aligned} A_\triangle &= \frac{1}{2} \times 6 \times 3\sqrt{3} \\ &= 9\sqrt{3} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{sector}} &= \frac{1}{2} \times (3\sqrt{3})^2 \times \frac{\pi}{3} \\ &= \frac{1}{2} \times 27 \times \frac{\pi}{3} \\ &= \frac{9\pi}{2} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore A_{\text{shaded}} &= A_\triangle - A_{\text{sector}} \\ &= 9\sqrt{3} - \frac{9\pi}{2} \text{ cm}^2 \end{aligned}$$

Question 23 (Sun) (2 marks)
- ✓ [1] for using the identity $\sec^2 x = \tan^2 x + 1$
- ✓ [1] for correct values of $\tan x$

$$\begin{aligned} \tan^2 x + \sec^2 x &= 9 \\ \tan^2 x + \tan^2 x + 1 &= 9 \\ 2\tan^2 x &= 8 \\ \tan^2 x &= 4 \\ \therefore \tan x &= \pm 2 \end{aligned}$$

Question 24 (Sun) (2 marks)
- ✓ [1] for substantial progress
- ✓ [1] for correct proof

$$\begin{aligned} \text{LHS} &= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta} \\ &= \frac{1}{\cos^2\theta\sin^2\theta} \\ &= \sec^2\theta\,\text{cosec}^2\,\theta \\ &= \text{RHS} \end{aligned}$$

Question 25 (Sun)

(a) (2 marks)
- ✓ [1] for any two correct values of $P(X = x)$
- ✓ [1] for all four correct values of $P(X = x)$

x	-3	-2	1	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(b) (2 marks)

✓ [1] for calculating $E(X)$ correctly

✓ [1] for final answer

$$\begin{aligned} E(X) &= -3 \times \frac{1}{8} - 2 \times \frac{3}{8} + 1 \times \frac{3}{8} + 3 \times \frac{1}{8} \\ &= -\frac{3}{8} \end{aligned}$$

$$\begin{aligned} \therefore \text{Expected loss after 1000 games} &= 1000 \times \frac{3}{8} \\ &= \$375 \end{aligned}$$

(c) (2 marks)

✓ [1] for simplification to $\dfrac{P(X > -2)}{P(X \geq -2)}$

✓ [1] for final answer

$$\begin{aligned} P(X > -2 | X \geq -2) &= \frac{P(X > -2 \cap X \geq -2)}{P(X \geq -2)} \\ &= \frac{P(X > -2)}{P(X \geq -2)} \\ &= \frac{\frac{4}{8}}{\frac{7}{8}} \\ &= \frac{4}{7} \end{aligned}$$

Question 26 (Lam)

(a) (2 marks)

- ✓ [1] for writing $\int_1^{14} \frac{k}{2t-1} dt = 1$
- ✓ [1] for correct proof

$$\begin{aligned} \int_1^{14} \frac{k}{2t-1} dt &= 1 \\ \frac{1}{2}k \int_1^{14} \frac{2}{2t-1} dt &= 1 \\ \frac{1}{2}k \Big[\ln(2t-1) \Big]_1^{14} &= 1 \\ \frac{1}{2}k (\ln 27 - \ln 1) &= 1 \\ \frac{1}{2}k \ln 27 &= 1 \\ \therefore k &= \frac{2}{\ln 27} \end{aligned}$$

(b) (2 marks)

- ✓ [1] for obtaining $\int_1^{t} \frac{k}{2t-1}\, dt = \frac{3}{4}$
- ✓ [1] for final answer

$$\begin{aligned} \int_1^{t} \frac{k}{2t-1}\, dt &= \frac{3}{4} \\ \frac{1}{2}k \Big[\ln(2t-1) \Big]_1^{t} &= \frac{3}{4} \\ \frac{1}{\ln 27} \Big[\ln(2t-1) \Big]_1^{t} &= \frac{3}{4} \\ \ln(2t-1) &= \frac{3\ln 27}{4} \\ 2t-1 &= e^{\frac{3\ln 27}{4}} \\ \therefore t &= 6.422... \end{aligned}$$

$\therefore$ After approximately 6.4 days.

Question 27 (Lam) (3 marks)

- ✓ [1] for any correct transformation
- ✓ [1] for any 3 correct transformations (incorrect order of transformations are ignored)
- ✓ [1] for correct sequence of transformations (correct order of transformations is required)

$$y = \frac{3}{-2\left(x - \frac{1}{2}\right)} + 5$$

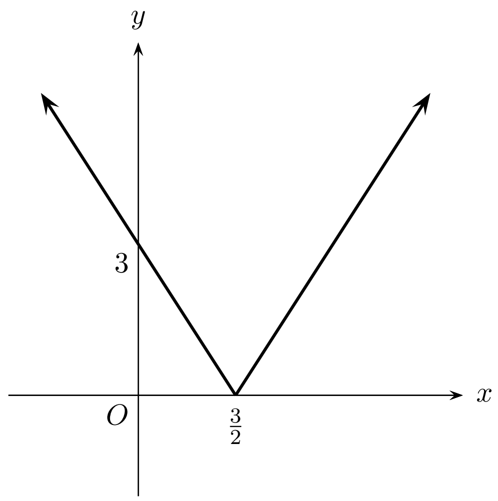
1. Reflect $y = \frac{1}{x}$ about the y-axis.
2. Dilate the resulting graph horizontally by a factor of $\frac{1}{2}$.
3. Translate the resulting graph horizontally to the right by $\frac{1}{2}$ units.
4. Dilate the resulting graph vertically by a factor of 3.
5. Translate the resulting graph vertically up by 5 units.

Other sequences of transformations are possible.

Question 28 (Lam)

(a) (2 marks)

- ✓ [1] for correct sketch
- ✓ [1] for correct axes intercepts



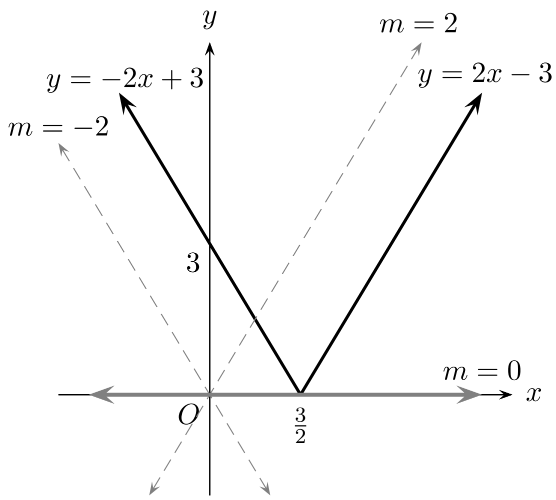
(b) (3 marks)

✓ [1] for substantial progress
✓ [1] for one correct value of x
✓ [1] for final answer

$$\begin{aligned}|6x-9| &= 3\\ 3|2x-3| &= 3\\ |2x-3| &= 1\\ 2x-3 &= \pm 1\\ 2x &= 3\pm 1\\ 2x &= 4, 2\\ x &= 2, 1\end{aligned}$$

(c) (1 mark)

✓ [1] for final answer



From the graph, $m < -2$, $m \geq 2$, or $m = 0$.

Question 29 (Lam) (4 marks)

✓ [1] for obtaining $f(g(x)) = \dfrac{1}{\sin^2 x + \sin x - 2}$
✓ [1] for letting $\sin^2 x + \sin x - 2 = 0$
✓ [1] for obtaining $\sin x = -2, 1$
✓ [1] for final answer

$$\begin{aligned}y &= f(g(x))\\ &= f(\sin x)\\ &= \frac{1}{\sin^2 x + \sin x - 2}\end{aligned}$$

$\therefore$ Vertical asymptotes of $y = f(g(x))$ occur at

$$\begin{aligned}\sin^2 x + \sin x - 2 &= 0\\ (\sin x + 2)(\sin x - 1) &= 0\\ \therefore \sin x &= -2, 1\\ \therefore \sin x &= 1 \text{ (as } \sin x \neq -2)\\ \therefore x &= \frac{\pi}{2}, \text{ for } x \in [0, 2\pi]\end{aligned}$$

Question 30 (Lam)

(a) (1 mark)

✓ [1] for final answer

$P(\text{Sophie wins on her first draw}) = \dfrac{4}{7}$.

(b) (2 marks)

✓ [1] for substantial progress
✓ [1] for final answer

$$\begin{aligned}&P(\text{Sophie wins in three or less of her turns})\\ =&P(\text{Sophie wins on her first draw})\\ &+ P(\text{Sophie wins on her second draw})\\ &+ P(\text{Sophie wins on her third draw})\\ =&\frac{4}{7} + \left(\frac{3}{7}\right)\left(\frac{4}{7}\right)\left(\frac{4}{7}\right) + \left(\frac{3}{7}\right)\left(\frac{4}{7}\right)\left(\frac{3}{7}\right)\left(\frac{4}{7}\right)\left(\frac{4}{7}\right)\\ =&\frac{4}{7} + \left(\frac{3}{7}\right)\left(\frac{4}{7}\right)^2 + \left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^3\\ =&\frac{4}{7}\left(1 + \frac{12}{49} + \left(\frac{12}{49}\right)^2\right)\\ =&0.7456...\end{aligned}$$

(c) (2 marks)

✓ [1] for establishing the correct infinite geometric series
✓ [1] for final answer

$$\begin{aligned}&P(\text{Sophie wins the game})\\ =&\frac{4}{7}\left(1 + \frac{12}{49} + \left(\frac{12}{49}\right)^2 + \cdots\right)\\ =&\frac{4}{7}\left(\frac{1}{1-\frac{12}{49}}\right)\\ =&\frac{28}{37}\end{aligned}$$

Question 31 (Lam) (3 marks)

✓ [1] for writing $A = \int_1^e (\ln(2x) - \ln x)\, dx$

✓ [1] for simplying the integrand using log laws

✓ [1] for final answer

$$\begin{aligned} A &= \int_1^e (\ln(2x) - \ln x)\, dx \\ &= \int_1^e \ln\left(\frac{2x}{x}\right) dx \\ &= \int_1^e \ln 2\, dx \\ &= \Big[x \ln 2\Big]_1^e \\ &= e\ln 2 - \ln 2 \\ &= (e-1)\ln 2 \ \text{ units}^2 \end{aligned}$$

Question 32 (Lam)

(a) (2 marks)

✓ [1] for obtaining correct expressions of $\sin\alpha$ AND d^2

✓ [1] for correct proof

$$I = \frac{k\sin\alpha}{d^2}$$
$$\sin\alpha = \frac{y}{d}$$
$$d^2 = y^2 + a^2$$

$$\begin{aligned} \therefore I &= \frac{k\left(\frac{y}{d}\right)}{y^2+a^2} \\ &= \frac{ky}{d\,(y^2+a^2)} \\ &= \frac{ky}{\sqrt{y^2+a^2}\,(y^2+a^2)} \\ &= \frac{ky}{(y^2+a^2)^{\frac{3}{2}}} \end{aligned}$$

(b) (3 marks)

✓ [1] for finding $\frac{dI}{dx}$, or equivalent merit

✓ [1] for obtaining $y = 0, \pm\frac{a}{\sqrt{2}}$

✓ [1] for proving that $y = \frac{a}{\sqrt{2}}$ provides the maximum illumination to the circumference

$$I = \frac{k^2y^2}{(y^2+a^2)^3}$$

$$\begin{aligned} \frac{dI}{dy} &= \frac{(y^2+a^2)^3 \cdot 2k^2y - k^2y^2 \cdot 3(y^2+a^2)^2 \cdot 2y}{(y^2+a^2)^6} \\ &= \frac{2k^2y(y^2+a^2)^3 - 6k^2y^3(y^2+a^2)^2}{(y^2+a^2)^6} \\ &= \frac{2k^2y(y^2+a^2)^2\left[(y^2+a^2) - 3y^2\right]}{(y^2+a^2)^6} \\ &= \frac{2k^2y(a^2-2y^2)}{(y^2+a^2)^4} \end{aligned}$$

Let $\frac{dI}{dy} = 0$

$$\therefore 2k^2y(a^2-2y^2) = 0$$
$$\therefore y = 0, \pm\frac{a}{\sqrt{2}}$$
$$\therefore y = 0, \frac{a}{\sqrt{2}} \ (\text{as } y \geq 0)$$

But $y = 0$ provides no illumination to the circumference.

$$\therefore y = \frac{a}{\sqrt{2}}$$

y	$\frac{a}{2}$	$\frac{a}{\sqrt{2}}$	a
$\frac{dI}{dy}$	$\frac{2k^2\left(\frac{a}{2}\right)\left(\frac{a^2}{2}\right)}{\left(\frac{a^2}{4}+a^2\right)^4}$	0	$\frac{2k^2a(-a^2)}{(a^2+a^2)^4}$
	$\nearrow$	$\longrightarrow$	$\searrow$

$\therefore y = \frac{a}{\sqrt{2}}$ provides the maximum illumination to the circumference.